Validation of an analytical model for estimating debris trajectories in a tornadic wind field

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SUMMARY:
Although the damage indicators in the EF scale cover a wide range of structural damage, it mostly does not consider the wind-induced movement of large compact objects such as vehicles, construction materials, and large appliances, which are often found in the aftermath of ground and drone surveys. Since there is no guidance in the EF-scale on the wind-induced movement of large compact objects, these observations are currently not being utilized to provide an estimate of the potential wind speeds. The objective of this paper is to provide an advanced analytical debris model for large compact objects in a tornadic wind field and validate the model by comparing these results to a wind tunnel study on vehicles performed by Haan et al. (2017). Overall, the analytical model shows good agreement for determining threshold sliding and lofting speeds of heavy compact objects and can be used to determine trajectories for lofted objects. In the future, this model will be applied to debris data obtained by the Northern Tornadoes Project to make recommendations for the application of this debris analysis method to supplement damage indicators found in the EF-scale.

Keywords: tornado, debris, aerial imagery

1. INTRODUCTION
The Enhanced Fujita (EF) Scale is currently used in Canada and the USA to rate the intensity of a tornado through observable damage indicators. These damage indicators provide varying degrees of damage, ranging in intensity from the threshold of visible damage, to total destruction. The degree of damage then provides the expected wind speed range that is associated with that level of damage (McDonald & Mehta, 2006). The overall EF-scale rating for a tornadic event is then assigned based on the maximum wind speed across all damage indicators. The current version of the Canadian EF-Scale uses 31 damage indicators which broadly covers tornadic damage done to homes, trees, commercial buildings, towers, transmission lines and farm structures (Sills et al., 2014). This is a modified version of the US EF-scale with slightly different wind speeds and six revised / additional damage indicators. Ground damage surveys in Canada can be challenging due to the low population density in much of the country, leading to sparse damage (and therefore, a lack of damage indicators) in many events. This lack of damage indicators in certain events has indicated a need for advanced analysis methods to help supplement damage indicators found in the EF-scale, such as the directional treefall method for estimating wind speeds in a tornado. Although the 31 damage indicators in the Canadian EF-scale covers a wide range of tornadic damage, it mostly does not consider the wind-induced movement of large compact objects. Large compact objects such as vehicles, construction materials, and large appliances are often found in
the aftermath of ground and drone surveys performed by the Northern Tornadoes Project (NTP). These surveys are then used to perform an engineering analysis on the damage in order to determine what type of damaging wind occurred and classify the event with an EF-scale rating (Sills et al., 2021). Since there is no guidance in the EF-scale on the wind-induced movement of large compact objects, these observations are currently not being utilized to provide an estimate of the potential wind speeds.

The objective of this paper is to provide an advanced analytical debris model for large compact objects in a tornadic wind field. The motivation for this paper is to compare these results to a wind tunnel study on vehicles performed by Haan et al. (2017), and eventually apply the validated model to the vast quantity of debris data obtained by the Northern Tornadoes Project in order to make recommendations for the application of this debris analysis method to supplement damage indicators found in the EF-scale.

2. METHODOLOGY
A tornado has a very complex structure due to its non-stationary, non-synoptic nature. Three-dimensional flow fields, non-linear effects, instabilities, and singularities need to be taken into account to properly model a tornado (Gillmeier et al., 2018). The development of model-scale tornado generators (Haan et al., 2008; Hangan and Kim, 2008) as well as collection of full-scale data (Refan et al., 2014) has added additional insight into the effect of tornado wind loads on the surface pressure of a building. Baker & Sterling (2017) used this available full-scale data for tornado and wind pressure fields to develop a simple analytical model to predict velocity time histories in order for debris trajectories to be calculated. The full derivation and discussion of this model can be found in Baker & Sterling (2017). Based on this derivation, the radial velocity normalized by the maximum radial velocity \((\bar{U} = U/U_m)\), the circumferential velocity normalized by the maximum radial velocity \((\bar{V} = V/U_m)\), and the vertical velocity normalized by the maximum radial velocity \((\bar{W} = W/U_m)\) can be expressed as:

\[
\bar{U} = \frac{-4\bar{r}\bar{z}}{(1 + \bar{r}^2)(1 + \bar{z}^2)} \quad (1)
\]

\[
\bar{V} = \frac{2.88\bar{r}[\ln(1 + \bar{z}^2)]}{(1 + \bar{r}^2)} \quad (2)
\]

\[
\bar{W} = \frac{4\delta \ln(1 + \bar{z}^2)}{(1 + \bar{r}^2)^2} \quad (3)
\]

where \(\bar{r}\) is the radial distance from the centre of the vortex \((r)\), normalized by the by the radial distance where the maximum radial velocity occurs \((r_m)\), \(\bar{z}\) is the vertical distance from the centre of the vortex \((z)\), normalized by the by the vertical distance where the maximum vertical velocity occurs \((z_m)\), \(S\) is the swirl ratio, defined as the maximum value of the circumferential velocity to the maximum value of the radial velocity \((S = U_m/V_m)\), and \(\delta\) is the ratio of the radial distance where the maximum radial velocity occurs to the vertical distance where the maximum vertical velocity occurs (which is a function of terrain roughness). The dynamics of windborne debris can be described through a set of differential equations of Newton’s 2nd law in a cylindrical coordinate system (radial, circumferential, and vertical directions), as shown in Figure 1.
Figure 1. Debris flight in a cylindrical coordinate system along with a cartesian coordinate reference.

\[
\frac{d^2 u}{dt^2} = \frac{\rho_a}{2\rho_m l} (C_D \cos \alpha - C_L \sin \alpha) [(U - u_m)^2 + (V - v_m)^2 + (W - w_m)^2] + \frac{v_m^2}{r} \tag{4}
\]

\[
\frac{d^2 v}{dt^2} = \frac{\rho_a}{2\rho_m l} (C_D \cos \beta - C_L \sin \beta) [(U - u_m)^2 + (V - v_m)^2 + (W - w_m)^2] \tag{5}
\]

\[
\frac{d^2 w}{dt^2} = \frac{\rho_a}{2\rho_m l} (C_D \cos \gamma + C_L \sin \gamma) [(U - u_m)^2 + (V - v_m)^2 + (W - w_m)^2] + \frac{\rho_a g}{\rho_m} - g \tag{6}
\]

where \(\frac{d^2 u}{dt^2}, \frac{d^2 v}{dt^2}\) and \(\frac{d^2 w}{dt^2}\) are the radial, circumferential, and vertical acceleration of a compact object, respectively, \(\rho_m\) is the density of the debris (assumed uniform through the object), \(l\) is the characteristic dimension of the object, \(C_D\) is the drag coefficient, \(C_L\) is the lift coefficient, \(U\), \(V\), and \(W\) are the radial, circumferential, and vertical wind speed, respectively, \(u_m\), \(v_m\), and \(w_m\) are the radial, circumferential, and vertical speeds of the object, respectively, and \(r\) is the radial distance from the center of the vortex. To validate this model, the results from this analysis will be compared to the results from Haan et al., (2017). Model-scale studies were performed both in a straight-line wind tunnel to determine the onset wind speeds for sliding, as well as a tornado vortex generator to determine the onset wind speeds for lofting for a 2002 Honda Odyssey minivan. During the testing, motion of the vehicle was restricted to ensure that flipping and lofting occur before sliding, allowing for a simplified analysis, and an easier direct comparison.

3. RESULTS
3.1. Straight-line wind tunnel comparison
Assuming that the straight-line wind is constant and is oriented in the radial direction, the equations of motion can be simplified, rather than using the full analytical model. If the vertical and circumferential wind speed are negligible, and the projectile does not move before it begins sliding, flipping, or lofting, the equation of motions for an initially stationary case can be expressed as:

\[
U_{\text{Sliding}} = \sqrt{\frac{2\mu_{ST} g \rho_m l}{\rho_a C_D}} \tag{7}
\]

Using the vehicle characteristics from Haan et al. (2017), there is a weak agreement (19.6\% percent difference) between Equation 7 and the wind tunnel study for the onset of sliding motion.
3.2. Tornado vortex generator comparison
To assess the threshold flight of a minivan, a lift coefficient of 0.25 was assumed based on the dimensions of the vehicle (Heisler, 2004). Figure 2 shows an example of an estimated trajectory of a lofted minivan calculated using Euler’s numerical method, based on the tornado characteristics from Haan et al, (2017). Overall, the minivan is estimated to loft at a low-end EF4 wind speed, which is in agreement with Haan et al. (2017).

![Figure 2. Example of a debris trajectory of a minivan in a 3D tornadic wind field.](image)

4. CONCLUSION
Overall, the analytical model shows good agreement for determining threshold sliding and lofting speeds of heavy compact objects and can be used to determine trajectories for lofted objects. In the future, this model will be applied to debris data obtained by the Northern Tornadoes Project to make recommendations for the application of this debris analysis method to supplement damage indicators found in the EF-scale.

REFERENCES